8.4 Exercise Set

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Solve.

1. *Car Trips.* During the first part of a trip, Jaclyn's Honda traveled 120 mi at a certain speed. Jaclyn then drove another 100 mi at a speed that was 10 mph slower. If the total time of Jaclyn's trip was 4 hr, what was her speed on each part of the trip? First part: 60 mph; second part: 50 mph

2. *Canoeing.* During the first part of a canoe trip, Terrell covered 60 km at a certain speed. He then traveled 24 km at a speed that was 4 km/h slower. If the total time for the trip was 8 hr, what was the speed on each part of the trip? First part: 12 km/h; second part: 8 km/h

3. *Car Trips.* Franklin's Ford travels 200 mi averaging a certain speed. If the car had gone 10 mph faster, the trip would have taken 1 hr less. Find Franklin's average speed. 40 mph

4. *Car Trips.* Mallory's Mazda travels 280 mi averaging a certain speed. If the car had gone 5 mph faster, the trip would have taken 1 hr less. Find Mallory's average speed. 35 mph

5. *Air Travel.* A Cessna flies 600 mi at a certain speed. A Beechcraft flies 1000 mi at a speed that is 50 mph faster, but takes 1 hr longer. Find the speed of each plane. Cessna: 150 mph, Beechcraft: 200 mph; or Cessna: 200 mph, Beechcraft: 250 mph

6. *Air Travel.* A turbo-jet flies 50 mph faster than a super-prop plane. If a turbo-jet goes 2000 mi in 3 hr less time than it takes the super-prop to go 2800 mi, find the speed of each plane. Super-prop: 350 mph; turbo-jet: 400 mph

7. *Bicycling.* Naoki bikes the 40 mi to Hillsboro averaging a certain speed. The return trip is made at a speed that is 6 mph slower. Total time for the round trip is 14 hr. Find Naoki's average speed on each part of the trip. To Hillsboro: 10 mph; return trip: 4 mph

8. *Car Speed.* On a sales trip, Jay drives the 600 mi to Richmond averaging a certain speed. The return trip is made at an average speed that is 10 mph slower. Total time for the round trip is 22 hr. Find Jay's average speed on each part of the trip. ⊡

9. *Navigation.* The Hudson River flows at a rate of 3 mph. A patrol boat travels 60 mi upriver and returns in a total time of 9 hr. What is the speed of the boat in still water? About 14 mph

10. *Navigation.* The current in a typical Mississippi River shipping route flows at a rate of 4 mph. In order for a barge to travel 24 mi upriver and then return in a total of 5 hr, approximately how fast must the barge be able to travel in still water? About 11 mph

11. *Filling a Pool.* A well and a spring are filling a swimming pool. Together, they can fill the pool in 4 hr. The well, working alone, can fill the pool in 6 hr less time than the spring. How long would the spring take, working alone, to fill the pool? 12 hr

12. *Filling a Tank.* Two pipes are connected to the same tank. Working together, they can fill the tank in 2 hr. The larger pipe, working alone, can fill the tank in 3 hr less time than the smaller one. How long would the smaller one take, working alone, to fill the tank? 6 hr

13. *Paddleboats.* Antonio paddles 1 mi upstream and 1 mi back in a total time of 1 hr. The speed of the river is 2 mph. Find the speed of Antonio's paddleboat in still water. About 3.24 mph

14. *Rowing.* Sydney rows 10 km upstream and 10 km back in a total time of 3 hr. The speed of the river is 5 km/h. Find Sydney's speed in still water. About 9.34 km/h

Solve each formula for the indicated letter. Assume that all variables represent nonnegative numbers.

15. $A = 4\pi r^2$, for r
(Surface area of a sphere of radius r) ⊡

16. $A = 6s^2$, for s
(Surface area of a cube with sides of length s) ⊡

17. $A = 2\pi r^2 + 2\pi rh$, for r
(Surface area of a right cylindrical solid with radius r and height h) $r = \dfrac{-\pi h + \sqrt{\pi^2 h^2 + 2\pi A}}{2\pi}$

⊡ Answers to Exercises 8, 15, and 16 are on p. IA-17.

18. $N = \frac{k^2 - 3k}{2}$, for k $k = \frac{3 + \sqrt{9 + 8N}}{2}$
(Number of diagonals of a polygon with k sides)

19. $F = \frac{Gm_1m_2}{r^2}$, for r
(Law of gravity) $r = \sqrt{\frac{Gm_1m_2}{F}}$, or $\frac{\sqrt{FGm_1m_2}}{F}$

20. $N = \frac{kQ_1Q_2}{s^2}$, for s $s = \sqrt{\frac{kQ_1Q_2}{N}}$, or $\frac{\sqrt{kQ_1Q_2N}}{N}$
(Number of phone calls between two cities)

21. $c = \sqrt{gH}$, for H
(Velocity of an ocean wave) $H = \frac{c^2}{g}$

22. $V = 3.5\sqrt{h}$, for h
(Distance to the horizon from a height) $h = \frac{V^2}{12.25}$

23. $w = \frac{lg^2}{800}$, for g
(An ancient fisherman's formula) $g = \sqrt{\frac{800w}{l}}$, or $\frac{20\sqrt{2lw}}{l}$

24. $V = \pi r^2 h$, for r
(Volume of a cylinder) $r = \sqrt{\frac{V}{\pi h}}$, or $\frac{\sqrt{V\pi h}}{\pi h}$

25. $a^2 + b^2 = c^2$, for b
(Pythagorean formula in two dimensions) $b = \sqrt{c^2 - a^2}$

26. $a^2 + b^2 + c^2 = d^2$, for c $c = \sqrt{d^2 - a^2 - b^2}$
(Pythagorean formula in three dimensions)

27. $s = v_0 t + \frac{gt^2}{2}$, for t
(A motion formula) $t = \frac{-v_0 + \sqrt{(v_0)^2 + 2gs}}{g}$

28. $A = \pi r^2 + \pi rs$, for r
(Surface area of a cone) $r = \frac{-\pi s + \sqrt{\pi^2 s^2 + 4\pi A}}{2\pi}$

29. $N = \frac{1}{2}(n^2 - n)$, for n
(Number of games if n teams play each other once) $n = \frac{1 + \sqrt{1 + 8N}}{2}$

30. $A = A_0(1 - r)^2$, for r
(A business formula) $r = 1 - \sqrt{\frac{A}{A_0}}$

31. $T = 2\pi\sqrt{\frac{l}{g}}$, for g
(A pendulum formula) $g = \frac{4\pi^2 l}{T^2}$

32. $W = \sqrt{\frac{1}{LC}}$, for L
(An electricity formula) $L = \frac{1}{W^2 C}$

Aha! 33. $at^2 + bt + c = 0$, for t
(An algebraic formula) $t = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

34. $A = P_1(1 + r)^2 + P_2(1 + r)$, for r
(Amount in an account when P_1 is invested for 2 years and P_2 for 1 year at interest rate r)
$r = -1 + \frac{-P_2 + \sqrt{(P_2)^2 + 4AP_1}}{2P_1}$

Solve.

35. *Falling Distance.* (Use $4.9t^2 + v_0 t = s$.)

a) A bolt falls off an airplane at an altitude of 500 m. Approximately how long does it take the bolt to reach the ground? 10.1 sec

b) A ball is thrown downward at a speed of 30 m/sec from an altitude of 500 m. Approximately how long does it take the ball to reach the ground? 7.49 sec

c) Approximately how far will an object fall in 5 sec, when thrown downward at an initial velocity of 30 m/sec from a plane? 272.5 m

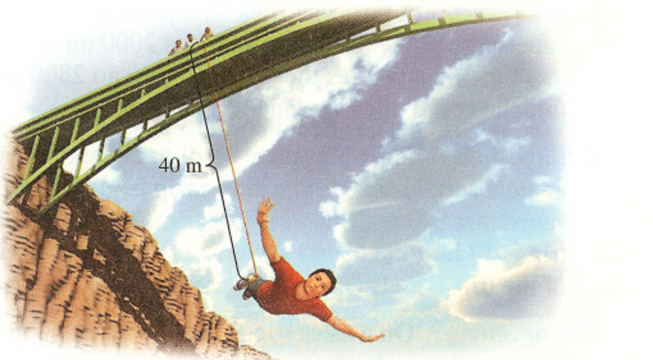
36. *Falling Distance.* (Use $4.9t^2 + v_0 t = s$.)

a) A ring is dropped from a helicopter at an altitude of 75 m. Approximately how long does it take the ring to reach the ground? 3.9 sec

b) A coin is tossed downward with an initial velocity of 30 m/sec from an altitude of 75 m. Approximately how long does it take the coin to reach the ground? 1.9 sec

c) Approximately how far will an object fall in 2 sec, if thrown downward at an initial velocity of 20 m/sec from a helicopter? 59.6 m

37. *Bungee Jumping.* Jaime is tied to one end of a 40-m elasticized (bungee) cord. The other end of the cord is tied to the middle of a bridge. If Jaime jumps off the bridge, for how long will he fall before the cord begins to stretch? (Use $4.9t^2 = s$.) 2.9 sec



38. *Bungee Jumping.* Mariah is tied to a bungee cord (see Exercise 37) and falls for 2.5 sec before her cord begins to stretch. How long is the bungee cord? 30.625 m

39. *Hang Time.* The NBA's LeBron James reportedly has a vertical leap of 44 in. What is his hang time? (Use $V = 48T^2$.) 0.957 sec
Source: www.vertcoach.com

40. *League Schedules.* In a bowling league, each team plays each of the other teams once. If a total of 66 games is played, how many teams are in the league? (See Exercise 29.) 12 teams

For Exercises 41 and 42, use $4.9t^2 + v_0t = s$.

41. *Downward Speed.* An object thrown downward from a 100-m cliff travels 51.6 m in 3 sec. What was the initial velocity of the object? 2.5 m/sec

42. *Downward Speed.* An object thrown downward from a 200-m cliff travels 91.2 m in 4 sec. What was the initial velocity of the object? 3.2 m/sec

For Exercises 43 and 44, use

$$A = P_1(1 + r)^2 + P_2(1 + r).$$

(See Exercise 34.)

43. *Compound Interest.* A firm invests $3000 in a savings account for 2 years. At the beginning of the second year, an additional $1700 is invested. If a total of $5253.70 is in the account at the end of the second year, what is the annual interest rate? 7%

44. *Compound Interest.* A business invests $10,000 in a savings account for 2 years. At the beginning of the second year, an additional $3500 is invested. If a total of $15,569.75 is in the account at the end of the second year, what is the annual interest rate? 8.5%

TW 45. Marti is tied to a bungee cord that is twice as long as the cord tied to Tivon. Will Marti's fall take twice as long as Tivon's before their cords begin to stretch? Why or why not? (See Exercises 37 and 38.)

TW 46. Under what circumstances would a negative value for t, time, have meaning?

SKILL REVIEW

To prepare for Section 8.5, review raising a power to a power and solving rational equations and radical equations (Sections 1.4, 6.4, 7.2, and 7.6).

Simplify.

47. $(m^{-1})^2$ [1.4] m^{-2}, or $\frac{1}{m^2}$

48. $(t^{1/3})^2$ [7.2] $t^{2/3}$

49. $(y^{1/6})^2$ [7.2] $y^{1/3}$

50. $(z^{1/4})^2$ [7.2] $z^{1/2}$

Solve.

51. $t^{-1} = \frac{1}{2}$ [6.4] 2

52. $x^{1/4} = 3$ [7.6] 81

SYNTHESIS

TW 53. Write a problem for a classmate to solve. Devise the problem so that (**a**) the solution is found after solving a rational equation and (**b**) the solution is "The express train travels 90 mph."

TW 54. In what ways do the motion problems in this section (like Example 1) differ from the motion problems in Section 6.5?

55. *Biochemistry.* The equation

$$A = 6.5 - \frac{20.4t}{t^2 + 36}$$

is used to calculate the acid level A in a person's blood t minutes after sugar is consumed. Solve for t. $t = \frac{-10.2 + 6\sqrt{-A^2 + 13A - 39.36}}{A - 6.5}$

56. *Special Relativity.* Einstein found that an object with initial mass m_0 and traveling velocity v has mass

$$m = \frac{m_0}{\sqrt{1 - \frac{v^2}{c^2}}},$$

where c is the speed of light. Solve the formula for c. $c = \frac{vm}{\sqrt{m^2 - (m_0)^2}}$

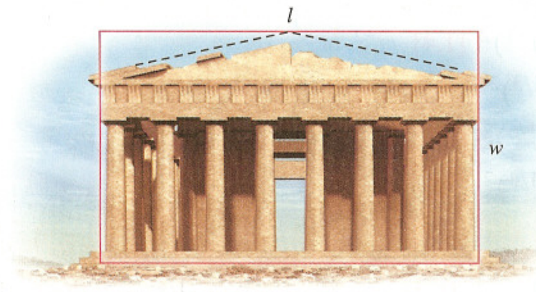
57. Find a number for which the reciprocal of 1 less than the number is the same as 1 more than the number. $\pm\sqrt{2}$

58. *Purchasing.* A discount store bought a quantity of paperback books for $250 and sold all but 15 at a profit of $3.50 per book. With the total amount received, the manager could buy 4 more than twice as many as were bought before. Find the cost per book. $2.50

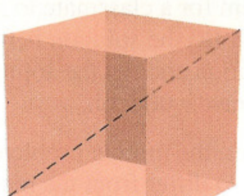
59. *Art and Aesthetics.* For over 2000 years, artists, sculptors, and architects have regarded the proportions of a "golden" rectangle as visually appealing. A rectangle of width w and length l is considered "golden" if

$$\frac{w}{l} = \frac{l}{w + l}. \qquad l = \frac{w + w\sqrt{5}}{2}$$

Solve for l.



60. *Diagonal of a Cube.* Find a formula that expresses the length of the three-dimensional diagonal of a cube as a function of the cube's surface area. $L(A) = \sqrt{\frac{A}{2}}$



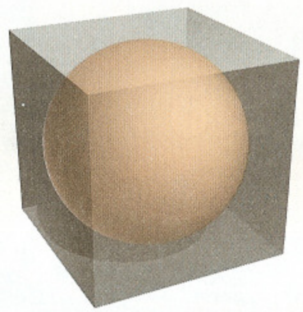
⊡ Answer to Exercise 62 is on p. IA-17.

61. Solve for n: $n = \pm\sqrt{\frac{r^2 \pm \sqrt{r^4 + 4m^4r^2p - 4mp}}{2m}}$

$$mn^4 - r^2pm^3 - r^2n^2 + p = 0.$$

62. *Surface Area.* Find a formula that expresses the diameter of a right cylindrical solid as a function of its surface area and its height. (See Exercise 17.) ⊡

63. A sphere is inscribed in a cube as shown in the figure below. Express the surface area of the sphere as a function of the surface area S of the cube. (See Exercise 15.) $A(S) = \frac{\pi S}{6}$



Try Exercise Answers: Section 8.4

1. First part: 60 mph; second part: 50 mph
15. $r = \frac{1}{2}\sqrt{\frac{A}{\pi}}$, or $\frac{\sqrt{A\pi}}{2\pi}$ **17.** $r = \frac{-\pi h + \sqrt{\pi^2h^2 + 2\pi A}}{2\pi}$
21. $H = \frac{c^2}{g}$

8.5 Equations Reducible to Quadratic

- Recognizing Equations in Quadratic Form
- Radical Equations and Rational Equations

RECOGNIZING EQUATIONS IN QUADRATIC FORM

Certain equations that are not really quadratic can be thought of in such a way that they can be solved as quadratic. For example, because the square of x^2 is x^4, the equation $x^4 - 9x^2 + 8 = 0$ is said to be "quadratic in x^2":

$$x^4 - 9x^2 + 8 = 0$$

$$(x^2)^2 - 9(x^2) + 8 = 0 \qquad \text{Thinking of } x^4 \text{ as } (x^2)^2$$

$$u^2 - 9u + 8 = 0. \qquad \text{To make this clearer, write } u \text{ instead of } x^2.$$

The equation $u^2 - 9u + 8 = 0$ can be solved by factoring or by the quadratic formula. Then, remembering that $u = x^2$, we can solve for x. Equations that can be solved like this are *reducible to quadratic* and are said to be *in quadratic form*.